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# RECURSIVE GATE-ARRAYS

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# Recursive Gate-Arrays<sup>1</sup>

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## Abstract

We propose a regular architecture, called *recursive gate-arrays*, suitable for circuits with modules of non-uniform size. A set of  $n$  (rectangular and L-shaped) modules can be placed in a recursive gate-array occupying  $\Theta(A_{\min})$  area, where  $A_{\min}$  is the sum of area of the modules. The placement can be obtained in  $O(n \log n)$  time.

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Design automation is motivated by the complexity of present day computing systems in the VLSI environment. The use of a regular structure facilitates the design process. In gate-array architecture a square region of the plane is partitioned into equal-sized square subregions and each module (functional cell) is placed in a distinct region.

Consider a circuit  $\mu$  which is a set of modules  $\{M_1, \dots, M_n\}$ , where  $n$  is assumed to be a perfect square. Let  $\alpha_i$  denote the area of module  $M_i$ . A gate-array layout of  $\mu$  requires  $\Omega(n\alpha_{\max})$  area, where  $\alpha_{\max} = \max_i \alpha_i$  is the area of the largest module of  $\mu$ . Thus, gate-arrays are suitable for circuits with "equal-sized" modules [CFKNS]. For circuits with non-uniform size modules the gate-array layout may result in excessive wastage of the layout area. In this paper, we propose a recursive-gate-array structure which requires  $\Theta(A_{\min})$  area to lay out an arbitrary circuit  $\mu$ , consisting of rectangular and L-shaped modules, where  $A_{\min} = \sum_i \alpha_i$ .

In a *recursive-gate-array* (RGA) architecture, a square region of the plane is partitioned into equal-sized square subregions. Each square subregion either contains a module or is itself an RGA (see Figure 1).

Layout of a graph  $G = (V, E)$ , in the unit-square-grid  $U = (P, L)$  (see [LP] for a formal definition of grids suitable for routing) is an embedding of  $G$  into  $U$ . Namely, each vertex  $v \in V$  is mapped to a grid-point  $p \in P$  and each edge  $e \in E$  is realized by a sequence  $(\ell_1, \dots, \ell_k)$  of grid-edges ( $\ell_i \in L$ ). We assume that each grid edge is used by at most one wire (i.e., using knock-knee layout mode where overlap of wires is not allowed [T]). Each module  $M_i$  is layout of a graph  $G_i$ ; boundary of each module is either rectangular or L-shaped. Area of each module is the area (measured in terms of grid units) enclosed by its boundary (see Figure 2).

If two modules correspond to the layout of the same graph then we say they can be *converted* to each other. If two modules with the same area (in an asymptotic sense) can

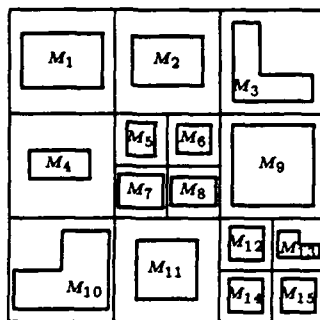


Figure 1: An RGA consisting of 15 modules

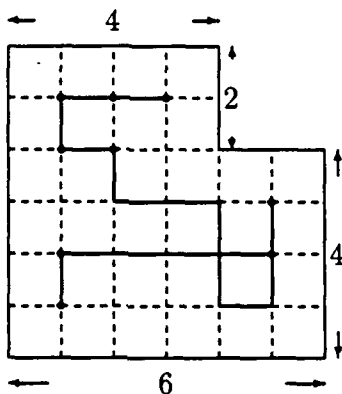


Figure 2: An L-shaped module with area 32

be converted to each other then they are called *equivalent*. Consider a module  $M$  occupying (grid) columns 1 thru  $\ell$ . If we remove all (horizontal) wires between column  $i$  and column  $i+1$  we obtain two modules: one with  $i$  columns and the other with  $\ell-i$  columns. We say  $M$  has been *partitioned* along  $x = i$ . Hereafter, we write  $M = (\omega, \ell)$  representing a rectangular module  $M$  with width  $\omega$  and length  $\ell$ ; by convention  $\omega \leq \ell$ . The next lemma shows a way to reconfigure rectangular modules without excessive wastage.

**Lemma 1** *There exists a module  $M' = (\omega', \ell')$  equivalent to a given module  $M = (\omega, \ell)$ , for  $\omega \leq \omega' \leq \sqrt{\omega\ell}$ .*

Proof: This result has been established in [L] for  $\omega' = \sqrt{\omega\ell}$ . Here, we prove the result for  $\omega \leq \omega' \leq \sqrt{\omega\ell}$ . Assume  $M$  occupies columns 1 thru  $\ell$ . We partition  $M$  along columns

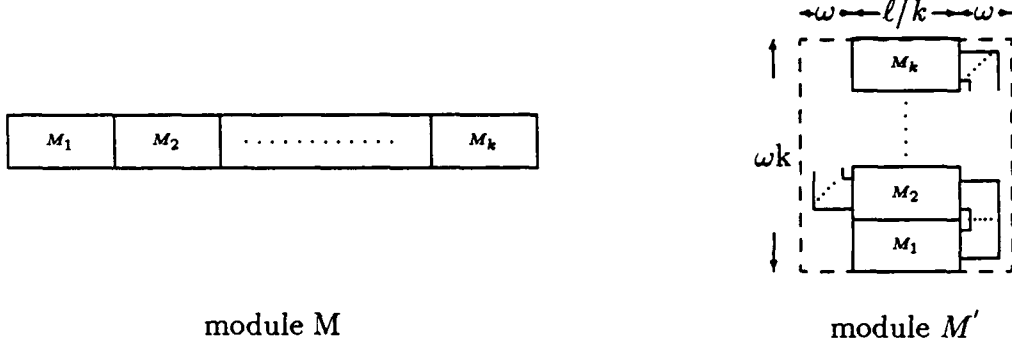


Figure 3: Converting a module

$x = \frac{\ell}{k}, x = 2\frac{\ell}{k}, \dots$ , and  $x = \frac{(k-1)\ell}{k}$  into  $k$  equal modules  $M_1, \dots, M_k$ , where  $M_i = (\omega, \frac{\ell}{k})$  for all  $i$ . We place the southwest corner of  $M_i$  at the grid point  $(1, \omega(i-1) + 1)$  and therefore its northeast corner at the grid point  $(\frac{\ell}{k}, \omega i)$  (see Figure 3). Next we interconnect  $M_i$  and  $M_{i+1}, 1 \leq i \leq k-1$ , as they were connected in  $M$ , that is, we add wires to realize the removed wires. The module  $M' = (\omega', \ell')$  is the rectangle enclosing all submodules and wires with  $\omega' = \omega k$  and  $\ell' = \frac{\ell}{k} + 2\omega$  (see Figure 3).

The area of  $M'$  is  $\alpha' = \omega' \ell' = \omega k (\frac{\ell}{k} + 2\omega) = \alpha + 2\omega^2 k$ , where  $\alpha$  is the area of  $M$ . Next, we establish a bound on the extra area used by this layout. By hypothesis we know  $\omega' = \omega k \leq \sqrt{\omega \ell} = \sqrt{\alpha}$  or  $\omega k \leq \sqrt{\alpha}$ . Since we have assumed  $\omega \leq \ell$  (we have),  $\omega \leq \sqrt{\alpha}$ . By combining these two we get  $\omega^2 k \leq \alpha$ , or equivalently,  $\alpha' \leq \alpha + 2\alpha$ . Since  $M'$  and  $M$  correspond to the same graph and  $\alpha' = \Theta(\alpha)$  then  $M'$  and  $M$  are equivalent.  $\square$

Using the previous lemma we can change the aspect ratio (width/length) of a rectangular module. Our placement technique consists of two phases. In the first phase we convert all

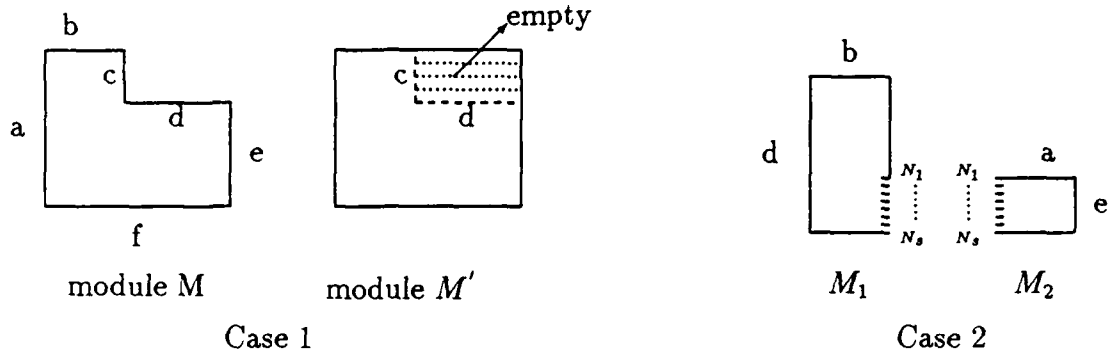


Figure 4: Converting an L-shaped module to a rectangular module

the modules into rectangles and in the second phase we reconfigure them appropriately.

Phase 1. Conversion to Rectangles

In this phase, all L-shaped modules are converted into rectangles. Let  $a, b, c, d, e$ , and  $f$  denote the six sides of an L-shaped module  $M$  in Figure 4, starting from the left side in a clockwise ordering. The area of  $M$  is denoted by  $\alpha$ .

*Case 1)* length of side  $e = \Theta(\sqrt{\alpha})$  and length of side  $b = \Theta(\sqrt{\alpha})$

Since the area of  $M$  is  $O(\alpha)$  (more precisely,  $\alpha$ ) then  $|c| = O(\sqrt{\alpha})$  and  $|d| = O(\sqrt{\alpha})$ , where  $|x|$  is length of side  $x$ . Let  $M'$  be the smallest rectangle enclosing  $M$ , and let  $\alpha'$  denote its area (see Figure 4). Note  $\alpha' = \alpha + |c||d| = \alpha + O(\alpha)$ . Therefore,  $\alpha' = O(\alpha)$ .

*Case 2)* length of side  $e = o(\sqrt{\alpha})$

We partition  $M$  (by cutting the nets  $N_1, \dots, N_s$ , where  $s \leq |e|$ ) into two rectangles  $M_1$  and  $M_2$ , where  $M_1 = (a, b)$  and  $M_2 = (e, d)$ . We denote areas of  $M_1$  and  $M_2$  by  $\alpha_1 (= ab)$  and  $\alpha_2 (= ed)$ , respectively. (see Figure 4)

We convert both  $M_1$  and  $M_2$  into squares  $M'_1$  and  $M'_2$  of sides  $a_1$  and  $a_2$ , respectively (see Lemma 1). Without loss of generality, assume  $a_1 \geq a_2$ . Note that  $a_1^2 = \Theta(\alpha)$ . We place  $M'_2$

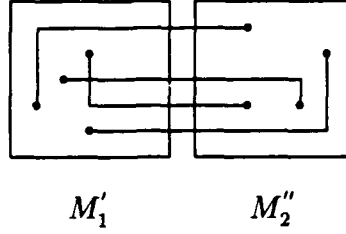


Figure 5: Interconnecting two sub-modules

in a square  $M''_2$  of side  $a_1$ . We place  $M'_1$  and  $M''_2$ , with total area  $\Theta(\alpha)$ , next to each other. Nets  $N_1, \dots, N_s$  are realized using  $s$  horizontal line and  $s$  vertical line. Thus each dimension is enlarged by  $s = o(\sqrt{\alpha})$ . (see Figure 5)

*Case 3)* length of side  $b = o(\sqrt{\alpha})$

Analysis is symmetric to Case 2.

Thus we can write the following lemma.

**Lemma 2** *An L-shaped module can be converted into an equivalent rectangular module.*

#### Phase 2. Placement

In this phase, first we convert all rectangles (output of Phase 1) into squares, as described in Lemma 1. We call the side of the smallest square one unit. Next we place each square in a square of side  $2^i$  units, for some integer  $i$ . Note that the length of the side of the latter square is at most twice the length of the side of the former one, and thus, its area is at most four times the area of the former one. We obtain a set of square modules  $\mu^* = \{M_1^*, \dots, M_n^*\}$ , where  $\alpha_i^*$  denote the area of  $M_i^*$ . By virtue of Lemma 1 and Lemma 2 we know  $A_{\min}^* = \Theta(A_{\min})$ , where  $A_{\min}^* = \sum_i \alpha_i^*$ .

We first place every four modules of side length  $2^i$  in a module of side length  $2^{i+1}$ . This



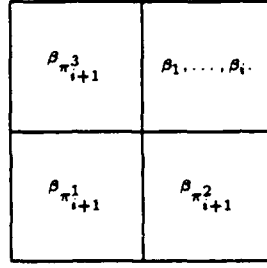


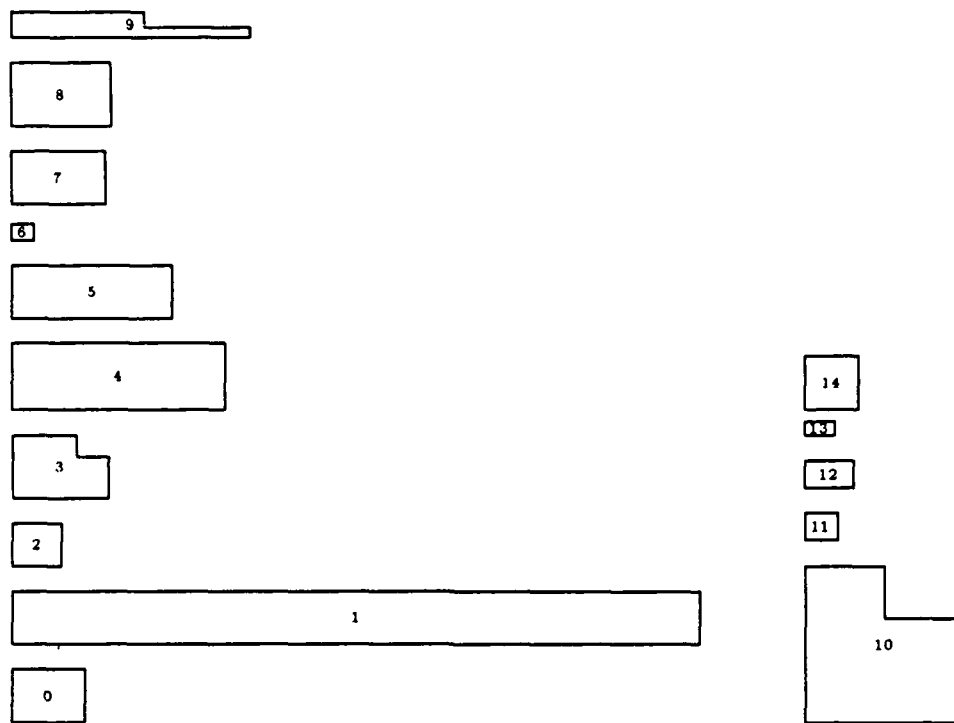
Figure 6: Recursive gate-array of  $\beta_1, \dots, \beta_{i+1}$

process is continued till there are no more than three modules of each size. This gives a set of blocks  $B_1, \dots, B_k$  each of which is a recursive gate-array. Note that the sum of areas of all the blocks  $B_i$  is  $A_{\min}^*$ . Next, we sort all the blocks with respect to their area and obtain an ordering  $\{(B_{\pi_1^1}, B_{\pi_1^2}, B_{\pi_1^3}), \dots, (B_{\pi_i^1}, B_{\pi_i^2}, B_{\pi_i^3})\}$ , where groups of three blocks are all of the same size. Note that some of  $B_{\pi_i^j}$ s may not exist. Let  $b_i$  be the side of  $B_{\pi_i^j}$ s and  $b_i < b_{i'}$  for  $i < i'$ . Let  $\beta_i$  denote the group of equal-sized blocks  $(B_{\pi_i^1}, B_{\pi_i^2}, B_{\pi_i^3})$ . Inductively assume that  $\beta_1, \dots, \beta_i$  have been placed in a recursive gate array of side  $2b_i$ . We can place  $\beta_1, \dots, \beta_{i+1}$  in a recursive gate-array of side  $2b_{i+1}$  as follows. Take a square of side  $2b_{i+1}$  and partition it into four squares of side  $b_{i+1}$ . Place  $B_{\pi_{i+1}^1}$ ,  $B_{\pi_{i+1}^2}$ , and  $B_{\pi_{i+1}^3}$  in three of the squares and place the recursive gate-array corresponding to  $\beta_1, \dots, \beta_i$ , with side  $2b_i$ , in the fourth square. This is always possible since  $2b_i \leq b_{i+1}$  (see Figure 6).

By induction, we conclude that  $\beta_1, \dots, \beta_t$  can be placed in a recursive gate-array of side  $2b_t$ . Since sum of the areas of  $B_i$ s is  $A_{\min}^*$  (or  $\Theta(A_{\min})$ ), and is greater than  $b_t^2$  (for there is at least one module with side  $b_t$ ) then  $(2b_t)^2 = \Theta(A_{\min})$ . Note that each (conversion) step takes  $O(1)$  time per module and sorting requires a total of  $O(n \log n)$  time. We conclude:

**Theorem -1** *A recursive gate-array layout of  $n$  modules, with minimum achievable area, can be obtained in  $O(n \log n)$  time.*

The proposed placement technique has been implemented. An example is given in Figure 7.



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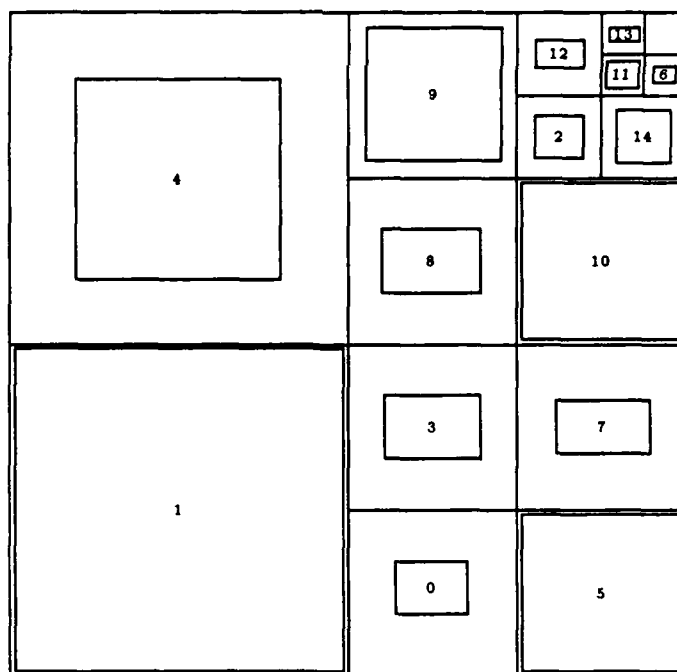


Figure 7: Demonstrating the proposed algorithm

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